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# A Note on the Generalized Inverted Exponential Software Reliability Model

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**Abstract**: In this paper we study the Hausdorff approximation of the Heaviside step function $h_r(t)$ by sigmoidal curve model based on the generalized inverted exponential software reliability model and find an expression for the error of the best approximation. Some comparisons are made.

**Keywords**: Generalized inverted exponential software reliability model, Hausdorff approximation, Heaviside step function, sigmoidal curve model

## I. INTRODUCTION

The generalized inverted exponential distribution is popular for modeling lifetime data in engineering, reliability, biomedical sciences and life testing [1]–[3]. Some software reliability models, can be found in [4]–[15]. A new class of Gompertz–type software reliability models and some deterministic reliability growth curves for software error detection, also approximation and modeling aspects, can be found in [17]–[19]. In this note we study the Hausdorff approximation of the Heaviside step function by sigmoidal curve model based on the generalized inverted exponential software reliability model and find an expression for the error of the best approximation.

## II. THE GENERALIZED INVERTED EXPONENTIAL SOFTWARE RELIABILITY MODEL

We consider the generalized inverted exponential cumulative distribution function – (GIECDF):

$$M(t;\theta,\phi) = \omega\left(1-\left(1-e^{-\frac{\theta}{t}}\right)^{\phi}\right). \tag{1}$$

We examine the special case $\omega = 1$, $t_0 = -\dfrac{\theta}{\ln\left(1-\left(\frac{1}{2}\right)^{\frac{1}{\phi}}\right)}$, i.e. $M(t_0;\theta,\phi) = \dfrac{1}{2}$.

The one–sided Hausdorff distance $d$ between the Heaviside step function

$$h_{t_0}(t) = \begin{cases} 0, & \text{if} \quad t < t_0, \\ [0,1], & \text{if} \quad t = t_0, \\ 1, & \text{if} \quad t > t_0, \end{cases} \tag{2}$$

and the sigmoid (1) satisfies the relation

$$M(t_0 + d;\theta,\phi) = 1 - d. \tag{3}$$

The following theorem gives upper and lower bounds for $d$

**Theorem**. Let

$$a = -\left(1+e^{-\frac{\theta}{t0}}\right)^{\phi}; \; b = 1 + \frac{e^{-\frac{\theta}{t_0}}\left(1-e^{-\frac{\theta}{t_0}}\right)^{\phi-1}\theta\phi}{t_0^2}$$

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For the one–sided Hausdorff distance $d$ between $h_{t_0}$ and the curve (1) the following inequalities hold for

$$\frac{1.5b}{-a} > e^{1.5},$$

$$d_l = \frac{1}{1.5\frac{b}{-a}} < d < \frac{\ln(1.5\frac{b}{-a})}{1.5\frac{b}{-a}} = d_r. \qquad (4)$$

**Proof.** Let us examine the functions:

$$F(d) = M(t_0 + d; \theta, \phi) - 1 + d. \qquad (5)$$

$$G(d) = a + bd. \qquad (6)$$

From Taylor expansion we obtain $G(d) - F(d) = O(d^2)$.

Hence $G(d)$ approximates $F(d)$ with $d \to 0$ as $O(d^2)$ (see Fig. 1).

In addition $G'(d) > 0$.

Further, for $\frac{1.5b}{-a} > e^{1.5}$ we have $G(d_l) < 0$ and $G(d_r) > 0$.

This completes the proof of the theorem.

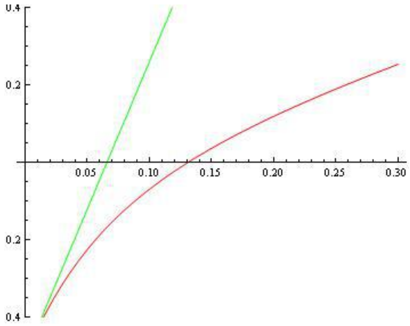


Fig. 1: The functions $F(d)$ and $G(d)$ for $\theta = 0.1$, $\phi = 2.1$.

The model (1) for $\theta = 0.2$, $\phi = 1.1$, $t_0 = 0.26302$ is visualized on Fig. 2.

The model (1) for $\theta = 0.1$, $\phi = 2.1$, $t_0 = 0.0788053$ is visualized on Fig. 3.

The model (1) for $\theta = 0.08$, $\phi = 2.9$, $t_0 = 0.0516678$ is visualized on Fig. 4.

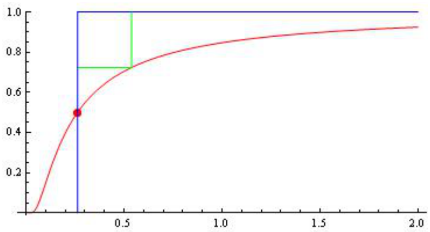


Fig. 2: The model (1) with $\theta = 0.2$, $\phi = 1.1$, $t_0 = 0.26302$; H–distance $d = 0.275813$.

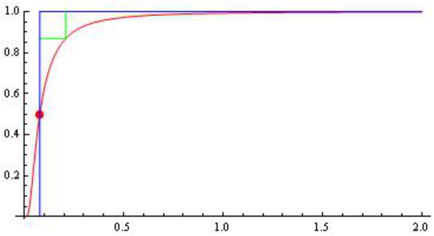


Fig. 3: The model (1) with $\theta = 0.1$, $\phi = 2.1$, $t_0 = 0.0788053$; H–distance $d = 0.130729$; $d_l = 0.0437909$; $d_r = 0.136992$.

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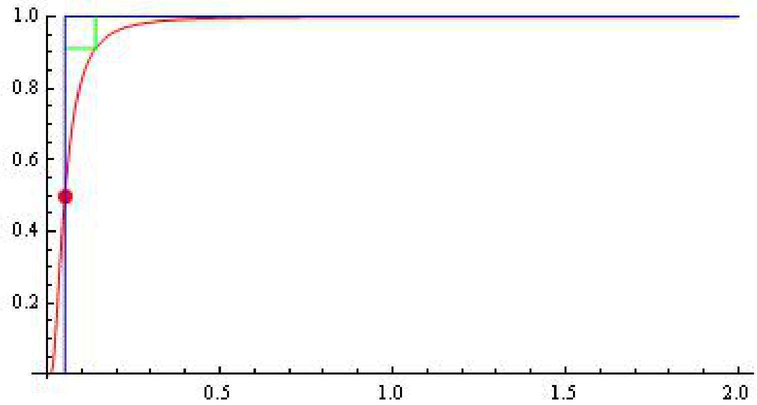


Fig. 4: The model (1) with $\theta = 0.08$, $\phi = 2.9$, $t_0 = 0.0516678$; H–distance $d = 0.0888784$; $d_l = 0.0261803$; $d_r = 0.0953682$.

## REMARKS

The estimation of remaining errors in the software is the deciding factor for the release of the software or the amount of more testing which is required software growth reliability models are using for the correct estimation of the remaining errors.

## NUMERICAL EXAMPLE

We examine the following data. (The data were reported by Musa [21] and represent the failures observed during system testing for 25 hours of CPU time).

**TABLE I**

FAILURES IN 1 HOUR (EXECUTION TIME) INTERVALS AND CUMULATIVE FAILURES [21], [20]

| Hour | Number of failures | Cumulative failures |
|---|---|---|
| 1 | 27 | 27 |
| 2 | 16 | 43 |
| 3 | 11 | 54 |
| 4 | 10 | 64 |
| 5 | 11 | 75 |
| 6 | 7 | 82 |
| 7 | 2 | 84 |
| 8 | 5 | 89 |
| 9 | 3 | 92 |
| 10 | 1 | 93 |
| 11 | 4 | 97 |
| 12 | 7 | 104 |
| 13 | 2 | 106 |
| 14 | 5 | 111 |
| 15 | 5 | 116 |
| 16 | 6 | 122 |
| 17 | 0 | 122 |
| 18 | 5 | 127 |
| 19 | 1 | 128 |
| 20 | 1 | 129 |
| 21 | 2 | 131 |
| 22 | 1 | 132 |
| 23 | 2 | 134 |
| 24 | 1 | 135 |
| 25 | 1 | 136 |

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```
f[t_] := 136 (1 - (1 - e^(-3.1446927524130714`/t))^1.1268337951971146`)
data1 = {{1, 27}, {2, 43}, {3, 54}, {4, 64}, {5, 75}, {6, 82}, {7, 84}, {8, 89}, {9, 92},
   {10, 93}, {11, 97}, {12, 104}, {13, 106}, {14, 111}, {15, 116}, {16, 122}, {17, 122},
   {18, 127}, {19, 128}, {20, 129}, {21, 131}, {22, 132}, {23, 134}, {24, 135}, {25, 136}};
d2 = Plot[f[t], {t, 0, 25}, PlotStyle → {Blue}, AspectRatio → 0.5, PlotRange → {0, 136}];
Show[d2, ListPlot[data1, Joined → True, Mesh → Full,
  MeshStyle → Directive[PointSize[Large], Thick]]]
```

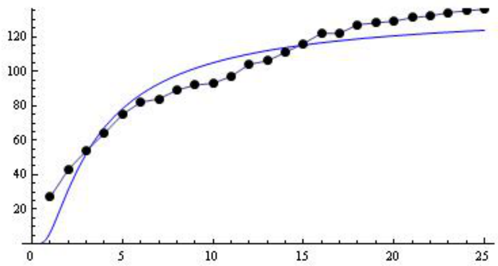


Fig. 5: Approximate solution.

The fitted model (1) based on the data of Table 1 for the estimated parameters:

$$\omega = 136; \ \theta = 3.1446927524; \ \phi = 1.1268337951$$

is plotted on Fig. 5.

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